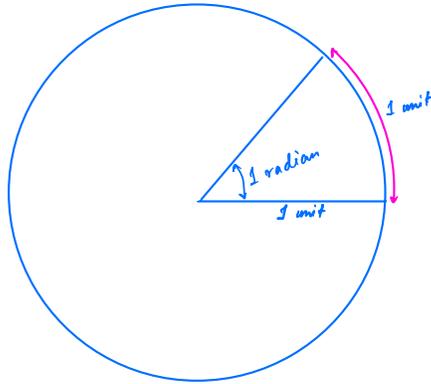


Radians: The trigonometric functions are measured with **radians**.

$$1 \text{ radian} = \frac{1 \text{ unit arc length}}{1 \text{ unit radius length}} \quad \left. \vphantom{\frac{1 \text{ unit arc length}}{1 \text{ unit radius length}}} \right\} \text{ for an unit circle.}$$



$$\begin{aligned} \text{Circumference} &= 2\pi \\ \text{Radius} &= 1. \end{aligned}$$

$$360^\circ = \frac{2\pi}{1} \text{ rad}$$

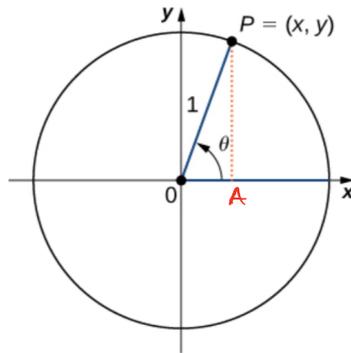
$$\Rightarrow 180^\circ = \pi \text{ rad}$$

$$1 \text{ radian} = 57.2958 \dots^\circ$$

Degree	Radian
0	0
30	$\frac{\pi}{6}$
45	$\frac{\pi}{4}$
60	$\frac{\pi}{3}$
90	$\frac{\pi}{2}$
120	$\frac{2\pi}{3}$
135	$\frac{3\pi}{4}$
150	$\frac{5\pi}{6}$
180	π

Basic Trigonometric Functions:

Let $P(x, y)$ be a point on a unit circle, centred at origin $(0, 0)$.



$$\begin{aligned} OA &= x = \text{Adjacent Side} \\ AP &= y = \text{Opposite Side} \\ OP &= \text{Hypotenuse} \end{aligned}$$

$$\begin{aligned} \triangle OAP \text{ is right angle} \\ \Downarrow \\ OA^2 + AP^2 = OP^2 \end{aligned}$$

Sine function

$$\sin \theta = \frac{\text{OPP}}{\text{HYP}} = \frac{y}{1} = y$$

Cosine function

$$\cos \theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{x}{1} = x$$

Tangent function

$$\tan \theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

not defined
for $x = 0$

Cosecant function

$$\csc \theta = \frac{\text{HYP}}{\text{OPP}} = \frac{1}{y} = \frac{1}{\sin \theta}$$

not defined
for $y=0$

Secant function

$$\sec \theta = \frac{\text{HYP}}{\text{ADJ}} = \frac{1}{x} = \frac{1}{\cos \theta}$$

not defined
for $x=0$

Cotangent function

$$\cot \theta = \frac{\text{ADJ}}{\text{OPP}} = \frac{x}{y} = \frac{1}{\tan \theta}$$

not defined
for $y=0$

Sin θ & Cos θ Value-table

θ	$\sin \theta$	$\cos \theta$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0

Pythagorean Identities:

$$\textcircled{1} \sin^2 \theta + \cos^2 \theta = 1$$

$$\textcircled{2} 1 + \tan^2 \theta = \sec^2 \theta$$

$$\textcircled{3} 1 + \cot^2 \theta = \csc^2 \theta$$

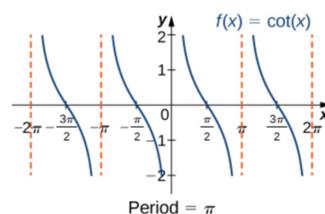
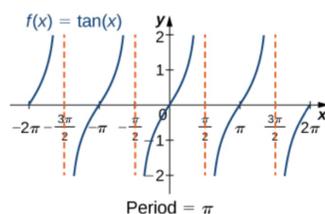
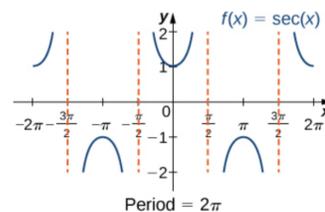
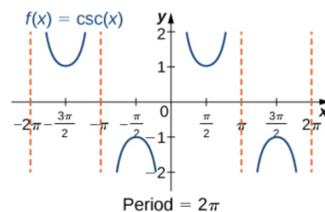
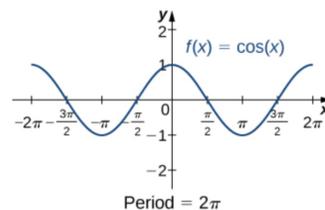
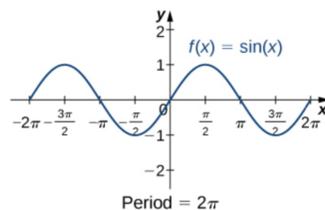
Addition/Subtraction formula:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta.$$

Cor: $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

$$\begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha \end{aligned}$$



A Particular form of trigonometric functions.

$$f(x) = A \sin(B(x-d)) + C$$

$d \rightarrow$ phase shift

$|A| \rightarrow$ Amplitude of f

$C \rightarrow$ Vertical shift

$B \rightarrow$ factor